Optimal Configuration of Sensors in Active Vibration Control

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Abstract

In this paper an investigation of a new formulation for active vibration control, which based on the optimal positions of piezoelectric sensors. The free vibration and modal properties are derived from the classical plate theory by using finite element method. A criterion is proposed to optimise the location of piezoelectric patches based on the observability Gramian of the structure and applied to a simply supported plate. The genetic algorithm is used to realise the optimal fitness function for finding the optimal configuration. The numerical simulation demonstrates that by locating piezoelectric sensors in the optimal positions, the energy delivery efficiency of the structure increases effectively.

***Keywords*: active vibration control, sensors, location, optimization.**

I. INTRODUCTION

Smart structures had investigated in numerous engineering across Aerospace, Energy and Marine sectors. These structures have the ability to adapt to environmental conditions according to the design requirements [1]. Some of these structures have low damping and stiffness, which can offer new modes of operation through active reconfiguration of their shape [2] and achieve energy harvesting using nonlinear stochastic resonance [3]. However, just as the flexible of the structures, control of structural vibration has been widely attracted by many researchers over recent year. Active control of some typical structures (e.g. a cylindrical shell) is investigated through using piezoelectric sensors and actuators [4]. Such active control is verified effective to suppress the vibration of structures through numerical simulations and experiments.

In recent years, the research focus on locations and numbers of actuators and sensors for active control has aroused interesting of many researchers in engineering [5]. The locations and numbers of actuators and sensors can directly affect the performance and efficiency of the active control. Misplaced sensors and actuators lead to problems such as lack of observability and controllability. A finite element model is developed based on Euler–Bernoulli beam theory to investigate active vibration control of beam structures with distributed sensor and actuator layers using a model-based linear quadratic regulator (LQR) controller [6]. Observability and controllability are also considered as an optimisation objective by using some optimisation criteria to ensure there is a good active control of structural vibration, and residual modes are always considered to limit the spillover effect [7].

This paper presents a modified optimisation criteria to determine the optimal location and numbers of sensors in active vibration based on observability. Meanwhile, the genetic algorithm (GA) code is used for the solution of the optimal problem. The sensors are assumed made of piezoelectric material. Then, the output energy is considered as an evaluation indicator compared with other configurations. The finite element method is used to assist the optimisation on the model of a simply support plate.

II. THE OPTIMIZATION CRITERIA FOR PIEZOELECTRIC SENSORS LOCATIONS

Locations and numbers of piezoelectric sensors are considered as optimal parameters in this paper. From the finite element analysis, equations of output energy by the sensors can be written as follows:

$$J = \int_0^{t_f} \boldsymbol{y}^T(t)\boldsymbol{y}(t)dt \tag{1}$$

where t is time and $\boldsymbol{y(t)}$ is displacement vector.

According to the method from Ref. [8], the equations of output energy can also be developed from Eq. (1) and expressed as

$$J = \boldsymbol{x}^T(0)\boldsymbol{W}\boldsymbol{x}(0) \tag{2}$$

where $\boldsymbol{W}$ is the observability gramian, defined by

$$\boldsymbol{W} = \int_0^{t} \boldsymbol{e^{At}C^T C e^{At}} dt \tag{3}$$

$\boldsymbol{A}$ and $\boldsymbol{C}$ are the dynamic equation from the system

$$\begin{cases} \dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t) \\ \boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{x}(t) \end{cases} \tag{4}$$

Equation (3) illustrate the observability gramian $\boldsymbol{W}$ will change with different locations of piezoelectric sensors. Therefore, the matrix norm of observability gramian can be used to maximise the output energy of the sensors. The corresponding observability gramian of each mode in linear systems can be expressed as

$$(W_O)_{ii} = (W_O^U)_{i+N,i+N} = \frac{1}{4\zeta_i\omega_i}\sum_{j=1}^{N_s}(c_{ij})^2 \tag{5}$$

and the gramian of each residual mode can be expressed as

$$(W_O)_{ii} = (W_O^R)_{i+N+N^R,i+N+N+N^R} = \frac{1}{4\zeta_i^R\omega_i^R}\sum_{j=1}^{N_R}(c_{ij}^R)^2 \tag{6}$$

where N is first eigenmodes and N^R residual eigenmodes; ω_i and ζ_i are the natural frequency and damping ratio of i^{th} the mode, and ω_i^R and ζ_i^R are those of the residual modes; c_{ij} is the sensing constant of the j^{th} sensor due to the motion of the i^{th} mode and c_{ij}^R is those due to the motion of the i^{th} residual mode. Consequently, if the i^{th} eigenvalue of W_O^U is small, it can cause that the i^{th} mode will not be observed well. In addition, the utilisation of residual modes can help to limit the spillover effects, because if the i^{th} eigenvalue of W_O^R is large, it can cause large spillover effects [9].

In order to obtain effective information about the N first eigenmodes and minimise the influence of each residual modes, an optimisation criterion is proposed to find the sensor locations:

$$J_{new} = max\{J_1 - \gamma J_2\} \tag{7}$$

$$J_1 = \sum W_O^U \cdot \min_{i=1,\cdots N} \frac{\left(W_O^U(S_1,\cdots S_{N_S})\right)_{ii}}{\max\limits_{S_1,\cdots S_{N_S}}\left(W_O^U(S_1,\cdots S_{N_S})\right)} \tag{8}$$

$$J_2 = \sum W_O^R \cdot \max_{i=1,\cdots N} \frac{\left(W_O^R(S_1,\cdots S_{N_S})\right)_{ii}}{\max\limits_{S_1,\cdots S_{N_S}}\left(W_O^R(S_1,\cdots S_{N_S})\right)} \tag{9}$$

where $S_1,\cdots S_{N_S}$ is the sensors' locations and N_S is the numbers of sensors, γ is a weighting constant. J_1 and J_2 represent the weighting output energy of the i^{th} first eigenmodes and the i^{th} residual eigenmodes, respectively, when the sensors are located in $S_1,\cdots S_{N_S}$. This new optimisation criteria utilize the output energy rate as a weighting term to program output energy of each mode, which using the unified evaluation standard to obtain optimal locations. The benefit of such criteria is taking into account the differences of output energy among different modes and improve the output energy rate.

III. The optimal numbers of piezoelectric sensors

Equation (3) shows that the observability depends on the configurations of sensors, which means an ideal configuration is that there is an individual sensor to observe each mode for to obtain the best observability. However, it is unrealistic to satisfy engineering application, therefore, we aim to seek for criteria of observability based on the criterion above, which can help to find suitable numbers of sensors [10]. A degree of observability for i^{th} mode is defined:

$$DEG_i = \frac{\sum_{j=1}^{N_s} c_{ji}^2}{\max\limits_{one\ sensor} c_{1i}^2} \tag{10}$$

The numerator of Eq. (10) is the output energy measured from piezoelectric sensors $(1\ldots N_s)$ for the i^{th} mode; the denominator is the maximal value of output energy for the i^{th} mode obtained by one sensor which is in the optimal location. When the value of DEG_i is over 100%, the i^{th} mode is better measured than it is specifically measured through using an optimally located sensor. Inversely, the objective is to minimize DEG_i the higher possible for residual modes. Then, this criterion is illustrated by a following numerical experiment.

IV. Results and discussion

In order to investigate the application of the proposed criteria, a simply supported plate is defined, equipped with N_s piezoelectric sensors to locate. The piezoelectric sensors are assumed to be perfectly attached on the surface of the plate with the small thickness, which can be ignored compared to the plate thickness. The dimensional and mechanical properties of the plate and piezoelectric sensors are listed in Tables 1.

TABLE I
Dimensional and mechanical properties of the plate and piezoelectric sensors

	Plate	PZT Sensor
$L\ (mm)$	1140	18
$W\ (mm)$	900	10
$H\ (mm)$	2	0.1
$E\ (Gpa)$	207	63
$\rho\ (kg/m^3)$	7870	7650
μ	0.292	0.30
e_{31}, e_{32}	-	-7.209
α, β	0.001	-

The plate is using Rayleigh damping for analysis, and the model is divided into 3600 rectangular elements, which correspond to the possible locations of actuators. The first six modes are assumed to be precisely measured while avoiding the residual modes: modes 7 and 8. The first eight mode shapes can be seen in Fig. 1.

The observability degree gives us information about the quality of the measure for each mode. It can also be used to define the numbers of sensors needed. When the smallest value DEG_i, in Fig. 2 and 3, is over 100%, it means that each mode can be better measured than when it is specifically measured by an actuator. Consequently, we can choose the value 100% as the criterion for the optimal numbers of sensors. Fig. 2 shows without considering the residual modes ($\gamma = 0$) the numbers of sensors needed is 3. In other words, 3 sensors can satisfy the value DEG_i larger than 100% for each first modes while relatively less for residual modes. This results can also be seen in Fig.4 that 3 sensors are suitable for observability with the consideration of residual modes ($\gamma = 1$).

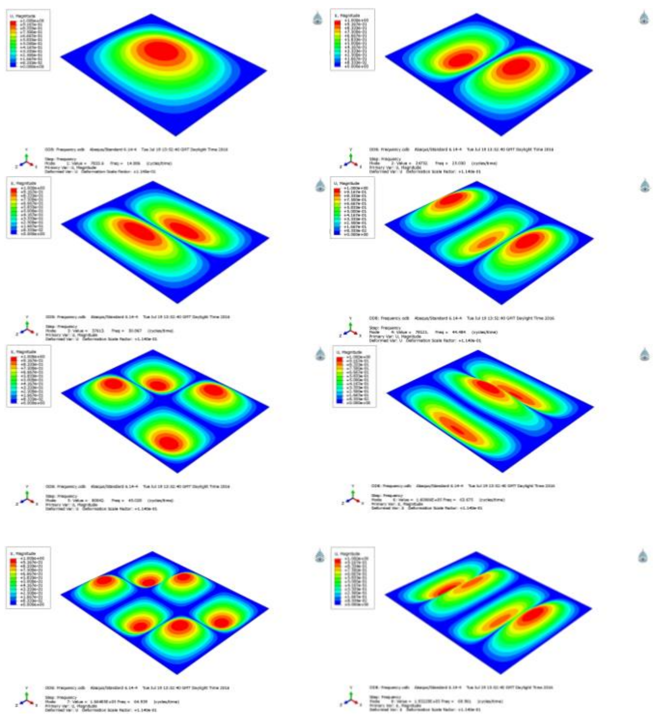

Fig. 1. Mode shapes of a simply supported plate.

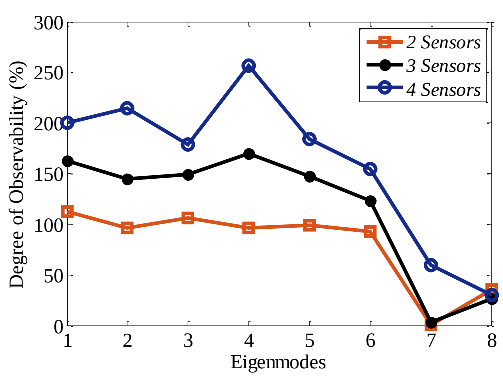

Fig. 2. DEG of sensors under new optimization criteria ($\gamma = 0$).

By comparing the results the different configuration, the results are shown in Fig. 4, which based on the I. Bruant's criteria [10], the new criteria in §.2 and a random configuration. Fig. 4 shows the location obtained from different criteria display a marked difference. Then, the strain energy is employed to evaluate these criteria.

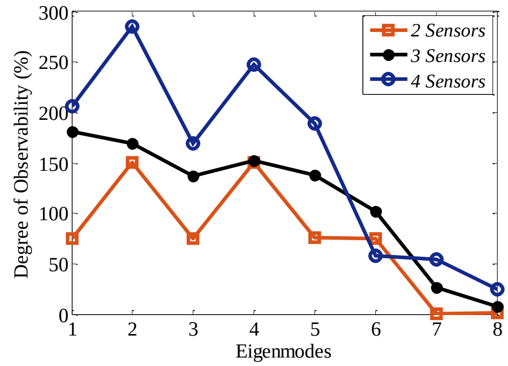

Fig. 3. DEG of sensors under new optimization criteria ($\gamma = 1$).

$$U = \frac{1}{2} V E \varepsilon^2 \propto \varepsilon^2 \tag{11}$$

where ε is strain, V is volume, and E is Young's modulus.

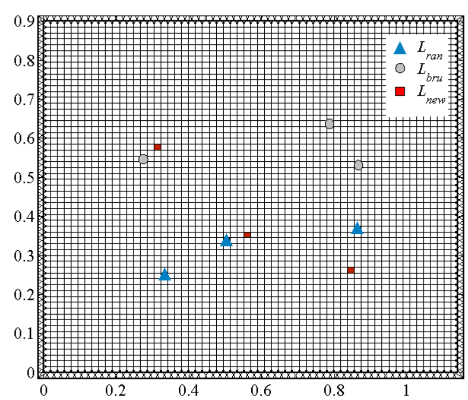

Fig. 4. Configuration results of three sensors based on different criteria.

Equation (11) is used to compare the value of output energy measured from the optimally located sensors. Fig. 5 shows that the new criteria in §.2 can obtain more output energy that the placement is more effective than the I. Bruant's criteria and random configuration.

V. Conclusion

In this paper, the problem of sensors locations and numbers is considered. A modified optimization criterion is proposed to optimize each problem, which can ensure good observability of each first modes of the structure and avoid overmuch observability of the residual modes. This optimization criterion can not only help to define the optimal locations of sensors but also the numbers. Its efficiency is shown by comparing them with other criteria, and the results show the optimization criteria is superior. Although the model used is simple, it can provide insight into the problem for real engineering applications.

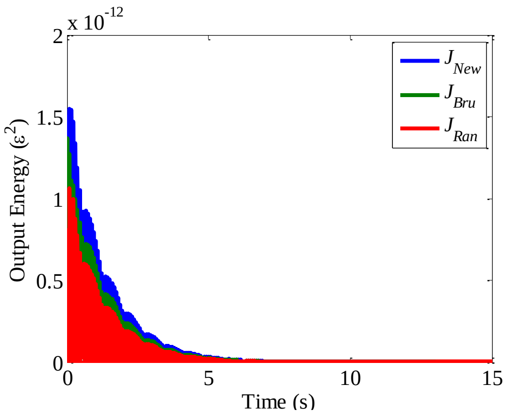

Fig. 5. Output energy comparison between different configurations.

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REFERENCES

[1] Preumont A and Seto K 2008 Active Control of Structures (John Wiley & Sons)

[2] Zhang J and McInnes C R 2015 Reconfiguring smart structures using approximate heteroclinic connections *Smart Mater. Struct.* 24 105034

[3] McInnes C R, Gorman D G and Cartmell M P 2008 Enhanced vibrational energy harvesting using nonlinear stochastic resonance *J. Sound Vib.* 318 655–62

[4] Qiu J and Tani J 1995 Vibration Control of a Cylindrical Shell Using Distributed Piezoelectric Sensors and Actuators *J. Intell. Mater. Syst. Struct.* 6 474–81

[5] Frecker M I 2003 Recent Advances in Optimization of Smart Structures and Actuators *J. Intell. Mater. Syst. Struct.* 14 207–16

[6] Ramesh Kumar K and Narayanan S 2008 Active vibration control of beams with optimal placement of piezoelectric sensor/actuator pairs *Smart Mater. Struct.* 17 055008

[7] Bruant I, Gallimard L and Nikoukar S 2010 Optimal piezoelectric actuator and sensor location for active vibration control, using genetic algorithm *J. Sound Vib.* 329 1615–35

[8] Hać a. and Liu L 1993 Sensor And Actuator Location In Motion Control Of Flexible Structures *J. Sound Vib.* 167 239–61

[9] Wang J, Yang Y, Zhang J and Su H 2012 Investigation of piezoelectric actuator optimal configuration for structural vibration control *Acta Aeronaut. Astronaut. Sin.* 33

[10] Bruant I 2005 Optimal Location of Actuators and Sensors in Active Vibration Control *J. Intell. Mater. Syst. Struct.* 16 197–206